

Production of K^- -mesons in proton-proton and proton-nucleus interactions at various energies

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ABSTRACT

The experimental data on the production of K^- -mesons in pp-collisions are analyzed and a method of the unified description of these data in a broad energy range for primary protons is proposed. The K^- -mesons production in pA-collisions is considered. The simple formulas for inclusive cross sections of the K^- production in these collisions are given. The results of the calculations by these formulas are compared with the available experimental data.

Introduction

Experimental investigations of the production of K^- -mesons in the nucleon-nucleon and nucleon-nucleus interactions have been carried out for a long time (see, for example, Refs. [1 – 14]). It has been found that the inclusive invariant cross section for the production of K^- -mesons in the pp-collisions rapidly acquires scaling character as a function of the transverse momentum, the radial scaling variable and the invariant energy [15, 16] as the energy increases. In a number of papers semiphenomenological analytical expressions have been suggested, which describe the experimental data on the K^- -mesons production in the pp-collisions [16, 17] and pA-collisions [18] at high energies. The direct experimental data on the total and differential cross sections for the K^- -mesons production in the pp-collisions at low energies are practically absent. In the papers [19 – 21] the expressions for the inclusive cross sections for the K^- -mesons production in these collisions near the threshold are given, which are based on the simple phenomenological models. However, in view of the development of high-current proton accelerators of a new generation – kaon factories – a need has arisen for a more detailed systematic description of the experimental information on the inclusive cross sections for the production of kaons in the nucleon-nucleon and nucleon-nucleus collisions in a broad energy range for primary protons. Such a

description in the case of the K^+ -mesons production in the pp- and pA-interactions has been considered in [22]. In this work an attempt of its generalization on the case of the K^- -mesons production in these interactions has been undertaken. Apart from the purely applied significance the obtained results may also serve the interpretation of the experiments [10, 23, 24] on the production of K^- -mesons in the nucleus-nucleus collisions at various energies.

1. The Total and Partial Cross Sections and Mean Multiplicities for the Production of K^- -Mesons in the pp Collisions

The experimental data on the production of K^- -mesons in the pp-collisions are available mainly in the region of high energies (at kinetic energy of a primary proton in the lab system $\epsilon_0 > 10$ GeV) in the form of mean multiplicities and inclusive spectra, while the data in the low-energy region (at $\epsilon_0 \leq 10$ GeV) are rather scarce. Let us consider at first the low-energy region. The experimental data on the total cross section for the K^- production in the pp-collisions, $\sigma_{pp \rightarrow K^- X}$ are practically absent at present. The authors know only one value of the cross section $\sigma_{pp \rightarrow K^- X}$ at $\epsilon_0 = 2.85$ GeV, which is equal to $4 \pm 2 \mu\text{b}$ [1]. This value is presented in Fig. 1. For the evaluation of $\sigma_{pp \rightarrow K^- X}$ at $\epsilon_0 \sim 3 \div 7$ GeV let us use the available data on the partial cross section for the production of K^- - and \bar{K}^0 -mesons in the pp-interactions. The reaction of the K^- production in these interactions, which has the lowest threshold ($\epsilon_{thr} = 2.49$ GeV) is the following reaction with four particles in the final state

$$p + p \rightarrow p + p + K^+ + K^-. \quad (1)$$

At present there is only one experimental value of the total cross section of this reaction $\sigma_{pp \rightarrow ppK^+K^-}$ at $\epsilon_0 = 5.7$ GeV, equal to $34 \pm 12 \mu\text{b}$, which is given in [19].*) Therein this cross section at $\epsilon_0 \leq 7$ GeV was calculated in the framework of one-kaon exchange model. The results of the calculation are represented in Fig. 1 by a dash-dotted curve. Beginning from the energies $\epsilon_0 \sim 5 \div 6$ GeV an appreciable contribution into the $\sigma_{pp \rightarrow K^- X}$ is given by the reactions with one and two additional π -mesons in the final state along with the reaction (1) [2, 26]. The existing experimental data on the total cross sections of these reactions are given in Table 1.

*)Note that the experimental investigation of the reactions of the production of $K\bar{K}$ -pairs in the pp-collisions in the energy range under consideration is planned at the accelerator CELSIUS (Uppsala, Sweden) in the near future [25].

**Table 1. Partial cross sections of the reaction $pp \rightarrow pNK^-K^0\nu\pi$,
 $\nu = 1, 2$ [2, 26]**

Reaction	ϵ_0 , GeV	ϵ_{thr} , GeV	σ , μb
$p + p \rightarrow p + p + K^- + K^0 + \pi^+ + \pi^0$	6.05	3.38	2.6 ± 2.5
$p + p \rightarrow p + p + K^- + K^0 + \pi^+$	6.99	2.94	13.8 ± 5.2
$p + p \rightarrow p + p + K^- + K^0 + \pi^+ + \pi^0$	6.99	3.38	5.9 ± 3.4
$p + p \rightarrow p + n + K^- + K^0 + 2\pi^+$	6.99	3.41	3.9 ± 2.8

From these data and from the data on $\sigma_{pp \rightarrow ppK^+K^-}$ presented in Fig. 1 it follows particularly that at $\epsilon_0 = 7$ GeV $\sigma_{pp \rightarrow K^-X} \approx 0.08$ mb. Note that at this energy the total cross section for the K^+ -mesons production $\sigma_{pp \rightarrow K^+X}$ constitutes a value of 0.9 mb [22]. To obtain an experimental estimate of the value of $\sigma_{pp \rightarrow K^-X}$ at $\epsilon_0 \sim 4 \div 5$ GeV let us suppose that $\sigma_{pp \rightarrow K^-X} = \sigma_{pp \rightarrow \bar{K}^0X}$ [21]. ^{†)} Using the sum of the partial cross sections of the following channels [26] ^{‡)}

$$\begin{aligned} p + p &\rightarrow p + p + K^0 + \bar{K}^0 \\ &\rightarrow p + n + K^+ + \bar{K}^0 \end{aligned} \quad (2)$$

as a value of $\sigma_{pp \rightarrow \bar{K}^0X}$ in this energy range we obtain that $\sigma_{pp \rightarrow K^-X} = 15.0 \pm 3.2 \mu\text{b}$ at $\epsilon_0 = 4.1$ GeV, while $\sigma_{pp \rightarrow K^-X} = 25.8 \pm 11.1 \mu\text{b}$ at $\epsilon_0 = 4.6$ GeV. These values are also presented in Fig. 1. Therein the dashed curve denotes the results of the calculation of $\sigma_{NN \rightarrow NNK\bar{K}}$ by the formula suggested in [21] on the basis of the experimental data analysis [26] on the total cross sections of the reactions (2) at $4.1 \leq \epsilon_0 \leq 9.1$ GeV.

$$\sigma_{NN \rightarrow NNK\bar{K}}(s) = \frac{1}{40} p_{max}^* \text{ mb}, \quad (3)$$

This formula was used in [21] as a total cross section for the production of K^- -mesons in the nucleon-nucleon collisions with four particles in the final state. Here

$$p_{max}^* = (2\sqrt{s})^{-1} (s - 4(m_N + m_K)^2)^{1/2} (s - 4m_N^2)^{1/2} \quad (4)$$

is the maximum momentum (in GeV/c) of \bar{K} -meson in the NN – c.m.s. for the reaction $NN \rightarrow NNK\bar{K}$; m_N and m_K are the rest masses of nucleon and kaon; and s is the invariant energy squared. It is seen that calculations by the one-kaon exchange model and by the formula (3), which, according to the above present, $\sigma_{pp \rightarrow K^-X}$ at $\epsilon_0 \leq 5$ GeV, do not agree with each other and with the experiment near the threshold (the arrow in Fig. 1). Thus, it is not correct to use these calculations for the above cross section in the energy range of $\epsilon_0 > 5$ GeV.

^{†)} It is easily seen that according to the hypothesis of the invariance of the strong interactions relative to the rotations in the isotopic space, the total cross sections for the production of K^- - and \bar{K}^0 -mesons in the pn-collisions are equal to each other [27].

^{‡)} One can neglect the contribution of the reactions with additional pions in the final state in $\sigma_{pp \rightarrow \bar{K}^0X}$ at $\epsilon_0 \sim 4 \div 5$ GeV [2, 26].

For the description of $\sigma_{pp \rightarrow K^- X}$ at $\epsilon_0 > 5$ GeV let us invoke some additional information, namely, the data on the mean multiplicity of K^- -mesons in the pp-collisions $\langle n_{K^-} \rangle$. The experimental information on $\langle n_{K^-} \rangle$ was obtained in [3] and listed in Table 2.

Table 2. Mean multiplicities of K^- -mesons in the pp- collisions

s, GeV ²	$\langle n_{K^-} \rangle$
25.3	0.008
37.8	0.036
46.8	0.033
67.2	0.07
81.0	0.08
100	0.11
133	0.13
485	0.24
960	0.29
2025	0.34
2810	0.37

Knowing the mean multiplicity $\langle n_{K^-} \rangle$, it is easy to find the total cross section for the K^- -meson production in the pp-collisions, since

$$\sigma_{pp \rightarrow K^- X} = \sigma_{pp}^{inel} \langle n_{K^-} \rangle. \quad (5)$$

Here σ_{pp}^{inel} is the inelastic pp-interaction cross section. The results of the calculation of the cross section $\sigma_{pp \rightarrow K^- X}$ by Eq. (5) is presented in Fig. 2.^{§)} It was supposed in this calculation that $\sigma_{pp}^{inel} = 30$ mb at $s < 100$ GeV² and $\sigma_{pp}^{inel} = 32$ mb at $s \geq 100$ GeV² [3, 28] and it was also considered that errors in $\langle n_{K^-} \rangle$ were equal to 25% at $67.2 \leq s \leq 133$ GeV² and 15% at the other values of s [3]. We shall seek the parameterization of these data in a form, such that it would become the expression given by the phase space model [29]:

$$\sigma_{pp \rightarrow K^- X} \sim (s - s_{min}^-)^{3.5}, \quad (6)$$

as $s \rightarrow s_{min}^-$ ($\sqrt{s_{min}^-} = 2(m_p + m_K) = 2.8639$ GeV), while at $s \rightarrow \infty$ it would satisfy the following relation [3, 30, 31] ¶)

$$\sigma_{pp \rightarrow K^- X}(s) = \sigma_{pp \rightarrow K^+ X}(s/s_0), \quad (7)$$

^{§)} The data on $\sigma_{pp \rightarrow K^- X}$ at low energy which are shown in Fig. 1 are also presented here.

¶) It is necessary to note that this relationship is actually true beginning from $s \sim 100$ GeV². It reflects the fact that the mean multiplicities of the particles of various type (π^\pm , K^\pm , \bar{p}) produced in the pp-collisions may be fitted by one universal curve, beginning from the Serpukhov's energies, if one simply redefines the gauge of the energy and changes the normalization [30, 31].

where $s_0 = 2.6 \pm 0.5 \text{ GeV}^2$ [30]. Taking into account that [22]

$$\begin{aligned}\sigma_{pp \rightarrow K^+ X}(s) &= \sigma_1^+ F_1(s/s_{min}^+) + \sigma_2^+ F_2(s/s_{min}^+), \\ F_1(x) &= (1 + 1/\sqrt{x}) \ln(x) - 4(1 - 1/\sqrt{x}), \\ F_2(x) &= 1 - (1/\sqrt{x})(1 + \ln(\sqrt{x}));\end{aligned}\tag{8}$$

$$\begin{aligned}F_1(x) &\approx (x-1)^3/3, \quad F_2(x) \approx (x-1)^2/8 \quad \text{for } x \rightarrow 1; \\ F_1 &\approx \ln(s), \quad F_2 \approx 1 \quad \text{for } s \rightarrow \infty; \\ \sigma_1^+ &= 2.8 \pm 0.8 \text{ mb}, \quad \sigma_2^+ = 9.7 \pm 1.5 \text{ mb}, \quad \sqrt{s_{min}^+} = m_p + m_{\Lambda^0} + m_K = 2.5476 \text{ GeV},\end{aligned}$$

and according to (6), (7) we choose the dependence of $\sigma_{pp \rightarrow K^- X}(s)$ to be the following form

$$\sigma_{pp \rightarrow K^- X}(s) = \left(1 - \frac{s_{min}^-}{s}\right)^k \left[\sigma_1^- F_1\left(\frac{s}{s_{min}^-}\right) + \sigma_2^- F_2\left(\frac{s}{s_{min}^-}\right) \right] + \sigma_3^- F_3\left(\frac{s}{s_{min}^-}\right), \tag{9}$$

$$F_3(x) = \left(\frac{x-1}{x^2}\right)^{3.5}, \quad \sigma_1^- = \sigma_1^+, \quad \sigma_2^- = \sigma_2^+ + \sigma_1^+ \ln\left(\frac{s_{min}^-}{s_{min}^+ s_0}\right) = 7.7 \pm 1.5 \text{ mb}.$$

The coefficients k and σ_3^- in (9), which have been found using the data presented in Fig. 2, are $k = 3$ and $\sigma_3^- = 3.90 \pm 0.85 \text{ mb}$. The calculation of the cross section $\sigma_{pp \rightarrow K^- X}$ according to (9) is represented in Figs. 1, 2 by the solid curve. It is seen that the approximation (9) well describes the existing^{||)} experimental data in a broad range of the primary proton energies. In this case, particularly, $\sigma_{pp \rightarrow K^- X} = 85 \mu\text{b}$ at $\epsilon_0 = 7 \text{ GeV}$.

2. The Invariant Inclusive Cross Section for the Production of K^- -Mesons in the pp Interactions

There are experimental data on the differential cross sections for the production of K^- -mesons in the pp-collisions only at $\epsilon_0 > 10 \text{ GeV}$. For the description of these data it is natural to choose the parameterization of the invariant inclusive cross section $E_{K^-} d\sigma_{pp \rightarrow K^- X}/d\mathbf{p}_{K^-}$ in the form used by us in [22] for the analogous cross section for the production of K^+ -mesons, namely:

$$E_{K^-} \frac{d\sigma_{pp \rightarrow K^- X}}{d\mathbf{p}_{K^-}} = B(s)(1 - x_R)^{n(s)} \exp \left[b - \sqrt{b^2 + c^2(s)p_1^2} \right], \tag{10}$$

$$x_R = \frac{E_{K^-}^*}{(E_{K^-}^*)_{max}} = \frac{2(E_0 + m_p)E_{K^-} - 2p_0 p_{K^-} \cos \vartheta}{s - (2m_p + m_K)^2 + m_K^2}, \tag{11}$$

^{||)} In the sense which was outlined above

where p_{\perp} is the transverse momentum of a K^{-} -meson, $E_{K^{-}}^{*}$ and $(E_{K^{-}}^{*})_{max}$ are the total energy of the K^{-} -meson in the centre-of-mass system and its maximum possible value, E_0 , $E_{K^{-}}$ and \mathbf{p}_0 , $\mathbf{p}_{K^{-}}$ are the energies and momenta in the lab system of a primary proton and kaon, respectively, and $\cos \vartheta = \mathbf{p}_0 \mathbf{p}_{K^{-}} / p_0 p_{K^{-}}$. From the comparison with the experimental data at the initial momenta $p_0 = 12.5$ GeV/c [4] and 24 GeV/c [5]; and from the threshold behaviour given by the phase space volume model and scaling behaviour [6, 15, 16] the following dependences of the parameters in Eq (10) on s were found:

$$B(s) = \left(200 + 2600 \exp \left[-\frac{(x-1)^2}{0.2} \right] - 340 \exp \left[-\frac{(x-0.6)^2}{0.03} \right] \right) \frac{\text{GeV } \mu b}{(\text{GeV}/c)^3 \text{ sr}},$$

$$n(s) = 2 + 3.8x, \quad b^2 = 3, \quad c^2(s) = 23x, \quad (12)$$

$$x = w / (0.3 + w + 0.63e^{-2w}), \quad w = (\sqrt{s/s_{min}} - 1)^2.$$

The comparison of the results of our calculations by Eqs. (10) – (12) for the double differential cross sections for the production of K^{-} -mesons in the pp-collisions with the experimental data [4 – 8] for $12.5 \leq p_0 \leq 1500$ GeV/c is given in Figs. 3 – 8. It can be seen that for $p_{\perp} \leq 1.5$ GeV/c there is a reasonable agreement of our calculations with the experiment in a broad energy range of a primary protons. It allows one to hope for the applicability of expressions (10) – (12) at any high energy. Moreover, since the parameters in the approximation (10) were obtained particularly from its threshold behaviour one can hope for the applicability of these expressions at low energies, taking into account the smooth dependence of $E_{K^{-}} d\sigma_{pp \rightarrow K^{-}X} / d\mathbf{p}_{K^{-}}$ on s . Naturally, in order to check directly the truth of the last assumption it is necessary to have the appropriate experimental data. The experimental information [9, 10] on the production of K^{-} -mesons in the collisions of protons with light nuclei (see the next section) may serve as another additional criterion for testing the applicability of formulas (10) – (12) at low energies. And finally, another important criterion for verification of the results obtained is the comparison of the total K^{-} -meson production cross sections found from the data on the multiplicity, partial cross sections for the production of \bar{K}^0 -mesons and from the double differential distributions. Therefore in Figs. 1 and 2 the dashed curve with two points represents our calculation of $\sigma_{pp \rightarrow K^{-}X}$ by the numerical integration of the expressions (10) – (12). It is evident that the cross section values thus found are close to those calculated by Eq. (9) and for $s > 20$ GeV² they even slightly underestimate them (by the value of the order of 30%).

Thus, the analysis carried out shows that formulas (9) for $\sigma_{pp \rightarrow K^{-}X}$ and (10) – (12) for $E_{K^{-}} d\sigma_{pp \rightarrow K^{-}X} / d\mathbf{p}_{K^{-}}$ describe the whole set of the experimental data on the production of K^{-} -mesons in the pp-collisions with an accuracy of 30 – 40%.

3. Production of K^- -Mesons in Proton-Nucleus Collisions

We consider at first the production of K^- -mesons in the collisions of protons with light nuclei. In such collisions the largest fraction of kaons is made up by those which are produced in direct proton-nucleon collisions, i.e. the production of kaons happens in the single event of the inelastic interaction of a primary proton with one of the intranuclear nucleons. The inclusive cross section for this process may approximately be represented as follows [22]:

$$E_{K^-} \frac{d\sigma_{pA \rightarrow K^- X}}{d\mathbf{p}_{K^-}} = I_V(A) E_{K^-} \frac{d\sigma_{pp \rightarrow K^- X}}{d\mathbf{p}_{K^-}}, \quad (13)$$

$$I_V(A) = A \int \rho(\mathbf{r}) d\mathbf{r} \exp \left[-\mu(p_0) \int_{-\infty}^0 \rho(\mathbf{r} + x' \mathbf{\Omega}_0) dx' - \mu(p_{K^-}) \int_0^{\infty} \rho(\mathbf{r} + x' \mathbf{\Omega}) dx' \right]; \quad (14)$$

$$\mu(p_0) = \sigma_{pp}^{inel}(p_0)Z + \sigma_{pn}^{inel}(p_0)N,$$

$$\mu(p_{K^-}) = \sigma_{K^-p}^{inel}(p_{K^-})Z + \sigma_{K^-n}^{inel}(p_{K^-})N, \quad (15)$$

where $\sigma_{K^-p}^{inel}$ ($\sigma_{K^-n}^{inel}$) is the inelastic free-particle cross section for the K^- -p (K^- -n) interaction, $\rho(\mathbf{r})$ is the average density of nuclear nucleons at a point \mathbf{r} , normalized to unity, Z ($N = A - Z$) is the number of protons (neutrons) in a nucleus, and $\mathbf{\Omega}_0 = \mathbf{p}_0/p_0$, $\mathbf{\Omega} = \mathbf{p}_{K^-}/p_{K^-}$. Here, the effect of the Fermi motion of nuclear nucleons on the kaon production in the collision of a primary proton with an intranuclear nucleon is neglected, since in the given region of initial energies, emission angles, and momenta of K^- -mesons, as shown by our calculations, its inclusion is unimportant. In addition, the K^- -meson production cross sections in the pp and pn-interactions are supposed to be the same [21, 27]. The inelastic cross sections of the proton-nucleon and kaon-nucleon interactions are taken to approximately describe the possibility of the K^- -meson production by a primary proton after proton's "soft" elastic rescatterings on the intranuclear nucleons and the chance of the "survival" of the kaon in a given interval of angle and momentum after kaon's drastically anisotropic [32] elastic rescatterings on the nucleons of the nucleus. Thus, in the framework of the single interaction model the knowledge of the cross section for the production of kaons in the nucleon-nucleon collisions allows one to calculate the cross section for kaons production in the nucleon-nucleus collisions according to the formulas (13) – (15). And, conversely, the experimental information about the production of kaons on light nuclei by the protons may be useful for the additional check-up of kaon production models in pp-collisions.

In the case where $\mathbf{\Omega} \approx \mathbf{\Omega}_0$, and for the Gaussian distribution of the density of nuclear nucleons ($\rho(\mathbf{r}) = (b_0/\pi)^{3/2} \exp(-b_0 r^2)$) we approximately have [22]

$$I_V(A) = A(1 - e^{-x_G})/x_G, \quad x_G = [\mu(p_0) + \mu(p_{K^-})] b_0/2\pi. \quad (16)$$

Note that in [22] the integral (14) has been calculated also for the nucleus with the uniform nucleon density and for $\mu(p_K) = 0$ it has been reduced to a more simple form.

In Fig. 9 we compare the results of our calculations by Eqs. (10) – (16) (solid curve) with the experimental data [9] for the inclusive invariant cross section for the K^- -meson production at an angle of 3.5° in the interaction of protons with $p_0 = 10.1$ GeV/c with Be nuclei. In the same place the results of calculations (dashed curve) are presented, where in (15) instead of the inelastic cross sections for the kaon-nucleon interaction $\sigma_{K^-N}^{inel}$ the total cross sections for this interaction $\sigma_{K^-N}^{tot}$ were used. The total cross sections for the inelastic proton-nucleon interaction and parameter b_0 hereafter were taken to be equal to 30 mb and 0.24 fm^{-2} , respectively [22]. The total and elastic ($\sigma_{K^-N}^{el}$) cross sections for the K^- -meson-nucleon interaction were borrowed from [33] and parametrized as follows

$$\begin{aligned}
\sigma_{K^-p}^{tot}(p_{K^-}) &= 22.6p_{K^-}^{-1.14}, & 0.245 \leq p_{K^-} \leq 0.7, \\
&= 47.54p_{K^-}^{0.94}, & 0.7 \leq p_{K^-} \leq 1.1, \\
&= 69.87p_{K^-}^{-3.1}, & 1.1 \leq p_{K^-} \leq 1.3, \\
&= 32.4p_{K^-}^{-0.17}, & 1.3 \leq p_{K^-} \leq 10, \\
&= 22, & 10 \leq p_{K^-} \leq 310, \\
\sigma_{K^-p}^{el}(p_{K^-}) &= 10.58p_{K^-}^{-0.98}, & 0.03 \leq p_{K^-} \leq 0.7, \\
&= 23p_{K^-}^{1.2}, & 0.7 \leq p_{K^-} \leq 1, \\
&= 23p_{K^-}^{-2.6}, & 1 \leq p_{K^-} \leq 1.5, \\
&= 9.56p_{K^-}^{-0.44}, & 1.5 \leq p_{K^-} \leq 20, \\
&= 2.56, & 20 \leq p_{K^-} \leq 310;
\end{aligned} \tag{17}$$

$$\begin{aligned}
\sigma_{K^-n}^{tot}(p_{K^-}) &= 36.7p_{K^-}^{0.75}, & 0.63 \leq p_{K^-} \leq 1, \\
&= 36.7p_{K^-}^{-0.726}, & 1 \leq p_{K^-} \leq 1.95, \\
&= 23.23p_{K^-}^{-0.05}, & 1.95 \leq p_{K^-} \leq 20, \\
&= 20, & 20 \leq p_{K^-} \leq 310, \\
\sigma_{K^-n}^{el}(p_{K^-}) &= 29.2p_{K^-}^{3.59}, & 0.612 \leq p_{K^-} \leq 0.9, \\
&= 17.1p_{K^-}^{-1.51}, & 0.9 \leq p_{K^-} \leq 3, \\
&= 3.25, & 3 \leq p_{K^-} \leq 310.
\end{aligned} \tag{18}$$

Here the momenta are measured in GeV/c and the cross sections are in mb. The inelastic cross sections for the K^- -meson-nucleon interaction were calculated as a difference between the total and elastic cross sections. It is seen that the calculation is in reasonable agreement with the experiment, although there is a small discrepancy in the softest part of the spectrum, which seems to be caused by the rescattering of kaons in the nuclear medium (which was taken into account only approximately in

deriving expression (13)), as well as by the contribution to this part of the spectrum of various channels (particularly of the channel $K^-N \rightarrow K^-N\nu\pi$, $\nu=1, 2\dots$) of the reaction $K^-N \rightarrow K^-X$ [33].

In Fig. 10 we show the energy dependence of the inclusive invariant cross section for the production of K^- -mesons with a momentum of 0.8 GeV/c and at an angle of 24° by protons on the ^{12}C nucleus compared with the experimental data [10]. It is evident that at $\epsilon_0 < 5$ GeV there is a satisfactory agreement of our calculations with the experiment. The discrepancy with the experiment at $\epsilon_0 > 5$ GeV as well as in the previous case seems to be due to the increasing contribution of inelastic K^-N -interactions to the cross section for K^- -mesons at higher energies in the considered energy range.

In Figs. 11 and 12 we present the comparison of our calculations of the double differential cross sections for the production of K^- -mesons by protons on Be nuclei with the experimental data [11, 12]: in Fig. 11 – by protons with momenta 18.8 (dashed curves) and 23.1 GeV/c (solid curves) at angles of 0 and 5.7° ; and in Fig. 12 – at $p_0 = 400$ GeV/c for K^- -mesons with $p_\perp = 0$ (solid curve) and 0.5 GeV/c (dashed curve). The absorption cross section of the primary proton by these nuclei in the calculations was assumed to be equal to 188 mb [22]. It is seen that there is a satisfactory agreement here as well.

Thus the results obtained show that the simple analytical formulas proposed (which are analogous to those obtained in [22] for K^+ -mesons) allow one to calculate satisfactorily the inclusive cross sections for the production of K^- -mesons at small angles on light nuclei by protons of various energies.

The calculation of the K^- -mesons production on heavy nuclei requires the inclusion of some additional factors, particularly of those which were outlined previously. However, as it follows from Refs. [13, 34], the experimental data on the kaon production cross section on heavy nuclei, at least in the region of the energies $\epsilon_0 = 24 \div 400$ GeV for $p_\perp \leq 1$ GeV/c, are approximately described by the dependence analogous to (13), where

$$\begin{aligned}
I_V(A) &= \left(\frac{A}{A_{Be}} \right)^{\alpha(x_F, p_\perp)} I_V(Be); \\
\alpha(x_F, p_\perp) &= \alpha_1(x_F) + \alpha_2(p_\perp), \\
\alpha_1(x_F) &= 0.74 - 0.55x_F + \frac{x_F}{|x_F|} 0.26x_F^2, \quad x_F = \vec{p}_\parallel^* / (\vec{p}_\parallel^*)_{max}, \\
\alpha_2(p_\perp) &= 0.1p_\perp^2.
\end{aligned} \tag{19}$$

This fact is obviously illustrated by Figs. 13 and 14. As one can see from these figures, the experiment is well fitted by the calculations.

Conclusion

The analysis carried out in this work showed that simple analytical formulas proposed for the inclusive cross sections of the production of K^- -mesons in the pp and pA-interactions satisfactorily describe the experimental data in a broad range of energies of primary protons. Therefore one can believe that the predictions for the cross sections, being obtained by their use, will appear to be sufficiently reliable and may be used in connection with the projects of the accelerators of a new generation – kaon factories.

We note, finally, that the maximum uncertainty in the data on the K^- -meson production in the nucleon-nucleon collisions refers to the energy region $\epsilon_0 < 10$ GeV. Therefore, for the better understanding of the kaon production phenomenon in the nucleon-nucleon collisions it is necessary to carry out the experimental measurements of the $\sigma_{pp \rightarrow K-X}$ and/or $E_K - d\sigma_{pp \rightarrow K-X}/d\mathbf{p}_{K^-}$ in this energy range.

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Figure Captions

Fig.1. Dependence of the total K^- -mesons production cross section on the energy in the pp-collisions. Calculations: the dot-dash line, the cross section $\sigma_{pp \rightarrow ppK^+K^-}$ calculated using the one-kaon exchange model [19]; the dashes, by (3); the solid curve, by (9); two dots and a dash, by the numerical integration of expressions (10)–(12). The experimental data: see text.

Fig.2. Dependence of the total K^- -mesons production cross section on the invariant energy squared s in the pp-collisions. The notations are the same as in Fig.1.

Fig.3. Double differential cross sections for the production of K^- -mesons, $d^2\sigma/dpd\Omega$, in the $p + p \rightarrow K^- + X$ reaction at $p_0 = 12.5$ GeV/c.

- a) Dependence of the $d^2\sigma/dpd\Omega$ on the longitudinal momentum \vec{p}_{\parallel}^* of a K^- -meson in the centre-of-mass system at $p_{\perp}^2 = 0.4$ (GeV/c)²;
- b) dependence of the $d^2\sigma/dpd\Omega$ on the p_{\perp}^2 at $p_{\parallel} = 0.6$ GeV/c. The experimental data: [4]. The solid curves, calculations by (10)–(12).

Fig.4. Inclusive invariant cross sections for the production of K^- -mesons in the pp-collisions at $p_0 = 24$ GeV/c as functions of $x = \vec{p}_{\parallel}^* / \vec{p}_{max}^*$ at fixed values (indicated by numbers by the curves) of their transverse momentum. The solid curves, the experimental data [5]. The dashed curves, calculations by (10)–(12).

Fig.5. Inclusive invariant cross section for the production of K^- -mesons in the pp-collisions at $p_0 = 24$ GeV/c as a function of their transverse momentum p_{\perp} at a fixed value of the variable x . The notations are the same as in Fig.4.

Fig.6. Inclusive invariant cross sections for the production of K^- -mesons in the pp-collisions at different values of the invariant energy squared s (indicated by numbers in the figure) as functions of their transverse momentum at fixed angles ϑ^* in the centre-of-mass system for given s ($\text{tg}\vartheta^* = 2\alpha/(s)^{1/2}$, $\alpha = 1.33$ GeV/c). The experimental data: [6]. The solid and dashed curves, our calculations by (10)–(12) at $s = 949$ and $s = 47$ GeV², respectively.

Fig.7. Inclusive invariant cross sections for the production of K^- -mesons in the pp-interactions at different collision energies $(s)^{1/2}$ (indicated by numbers in the figure) as functions of $x_F = 2 \vec{p}_{\parallel}^* / (s)^{1/2}$ at fixed values of their transverse momentum p_{\perp} . The experimental data: [8]. The solid curves, calculations by (10)–(12) at $(s)^{1/2} = 53$ GeV.

Fig.8. Inclusive invariant cross section for the production of K^- -mesons in the

pp-interactions at different collision energies $(s)^{1/2}$ as a function of their transverse momentum p_{\perp} at a fixed value of the variable x_F' . The notations are the same as in Fig.7.

Fig.9. Inclusive invariant cross section for the production of K^- -mesons at an angle of 3.5° in the interaction of protons of momentum 10.1 GeV/c with Be nuclei as a function of the K^- -meson momentum.

Fig.10. Inclusive invariant cross section for the production of K^- -mesons with momentum of 0.8 GeV/c at an angle of 24° on ^{12}C nuclei as a function of primary-proton energy. The solid and dashed curves denote the same as in Fig.9. The dot-dashed curve denotes the same as dashed curve, but it is supposed that the K^- -meson production cross section in the pn-collisions is larger than one in the pp-collisions by a factor of two [20].

Fig.11. Double differential cross sections for the production of K^- -mesons at angles of 0 and 5.7° in the interaction of protons of momenta 18.8 and 23.1 GeV/c with Be nuclei as functions of the K^- -meson momentum.

Fig.12. Double differential cross sections for the production of K^- -mesons per proton, that interacted with Be nuclei, with $p_0 = 400$ GeV/c at $p_{\perp} = 0$ and 0.5 GeV/c as functions of the K^- -meson momentum.

Fig.13. Spectra of K^- -mesons per proton, that interacted with Cu nuclei, with $p_0 = 24$ GeV/c at angles of 57 and 87 mrad. The experimental data: [14]. The solid and dashed curves, our calculation by (10)–(19), respectively, for angles of 57 and 87 mrad and $\sigma_a = 850$ mb.

Fig.14. Dependence of the inclusive invariant cross sections for the production of K^- -mesons in the collisions of protons with $p_0 = 100$ GeV/c with Cu and Pb nuclei on their momentum at $p_{\perp} = 0.3$ GeV/c. The experimental data: [13]. The dashed and solid curves, our calculation by (10)–(19), respectively, for Cu and Pb nuclei.